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## Abstract

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Improving the signal to noise ratio may be possible considering the power line as a two port network, with input and output impedances and also considering the output impedance of the noise.

Not only the voltage signal injected in the line but also the current is seems to be limited. Further work needs to be done to determine which are important in each case. But in general adapting the injected signal to the input impedance on the line seem may be important.

Adaptive impedances can be simulated at the receiver, but the signal to noise ratio at the receiver is independent of the receiver impedance as long as the system is linear. Low impedances in cases where the signal level is high may improve the signal to noise ratio if the increase of the received current increases the output impedance of the noise sources.

# Models for the power line channel

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In a transmission line the amplitude and power of the signal can decrease due to the losses in the dielectric (since it is not a perfect insulator). Losses in the resistance of the conductor since (the current is not zero because the receiver impedance is adapted to the line or low to reduce noise) may be accounted for as a resistive divisor should be lower as long as the receiver impedance is not too low.

The resistance of a 1mm, 100m length copper power line is 8  $\Omega$  at 1 MHz and 37  $\Omega$  at 20 MHz. This should give an idea of the values of the receiver impedance.

## *Resistance of a transmission line due to the skin effect*

The electric field in a conductor has the form,

$$E(z) = E e^{-\frac{z}{\delta}}$$

The current density will be given by,

$$J(z) = \sigma E(z)$$

And the resulting current will be given by,

$$I = \iint J(z) = E \sigma \delta W$$

where  $W$  is the conductor width. For a conductor of length  $L$  one has  $E=V/L$  ( $E$  is the electric field at the surface of the conductor, where the voltage is applied). One can now make,

$$R = \frac{L}{\sigma \delta W}$$

with,

$$\delta = \frac{1}{\sqrt{\frac{\omega \mu \sigma}{2}}}$$

The resistance grows with the root the frequency.

The resistivity of copper is, 0.16778523 mOhm mm<sup>2</sup>/cm. For a 1mm cable,  $W$  will be given by the perimeter of the cable,  $2\pi d/2$ . The magnetic permeability for copper is the same as for air,  $\mu = \pi/25000000$  A/m<sup>2</sup>. Assuming a cable of 100 m, and or a 1 MHz frequency one has,

$$\delta = 65,1924 \mu m$$

$$R = 8,19232 \Omega$$

For a 20 MHz frequency one was,

$$\delta = 14,5775 \mu m$$

$$R = 36,6372 \Omega$$

### *Transmission line equations*

In a line with perfect conductors immersed in low loss dielectric one has:

The loss angle is:

$$\tan(\theta) = \frac{\sigma}{\omega \varepsilon} \Leftrightarrow \theta \approx \frac{\sigma}{\omega \varepsilon}$$

The propagation constant is,

$$k_z = \omega \sqrt{\mu \varepsilon (1 - j\theta)} \Leftrightarrow k_z = \sqrt{\mu \omega (\omega \varepsilon - j\sigma)}$$

These losses are independent of the current value and are not related with conductor resistance.

Most important equations of a transmission line:

Propagation constant

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = j\beta = j\omega \sqrt{LC}$$

Characteristic Impedance of the wave

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_0 = R_0 = \sqrt{\frac{L}{C}}$$

Input Impedance

$$Z_{in} = Z_0 \frac{\frac{Z_L}{Z_0} + \tanh[\gamma l]}{1 + \frac{Z_L}{Z_0} \tanh[\gamma l]}$$

$$Z_{in} = Z_0 \frac{\frac{Z_L}{Z_0} + j \tan[\beta l]}{1 + j \frac{Z_L}{Z_0} \tan[\beta l]}$$

**This impedance oscillates between  $Z_L$  and  $Z_0^2 / Z_L$ .** For small lines,

$$Z_{in} = \frac{Z_L + j\omega L}{1 + j\omega C Z_L}$$

for  $Z_L \ll Z_0$  this is approximately given by,

$$Z_{in} = Z_L + Z_0^2 / Z_L$$

This represents a low pass filter.

Open Terminal Impedance

$$Z_{l0} = Z_0 \coth[\gamma l]$$

$$Z_{l0} = -j Z_0 \cot[\beta l]$$

Transmission Equations

$$V_y = V_2 \cosh[\gamma y] + Z_0 I_2 \sinh[\gamma y]$$

$$I_y = V_2/Z_0 \sinh[\gamma y] + I_2 \cosh[\gamma y]$$

$$V_y = V_2 \cos[\beta y] + j Z_0 I_2 \sin[\beta y]$$

$$I_y = j V_2/Z_0 \sin[\beta y] + I_2 \cos[\beta y]$$

$$V_y = V_2 \cos[\beta y] + V_a \sin[\beta y]$$

$$I_y = j (V_2/Z_0 \sin[\beta y] - V_a/Z_0 \cos[\beta y])$$

$$V_{c1} = \frac{V_2 + V_a j}{2}$$

$$V_{c2} = \frac{V_2 - V_a j}{2}$$

$$V_y = e^{jy\beta} V_{c2} + e^{-jy\beta} V_{c1}$$

$$I_y = \frac{e^{jy\beta} V_{c2}}{Z_0} - \frac{e^{-jy\beta} V_{c1}}{Z_0}$$

## Two port models for the power line channel

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The equations for the travelling waves and for the frontier conditions are,

$$K_{pp} = e^{-k l i - \alpha l}, K_{pn} = e^{+k l i - \alpha l}$$

$$V_1 = A K_{pp} + B K_{pn}$$

$$V_0 = A + B, I_0 = \frac{A}{Z} - \frac{B}{Z}$$

Where,  $l$ , is the line length,  $k$ , is the wave number,  $\alpha$ , is the attenuation factor  $Z$  is the characteristic impedance of the line,  $V_0$ ,  $I_0$ ,  $V_1$  and  $I_1$  are the voltage and current at the near and far end of the line. The current represents the current entering the line and so  $I_0$  and  $I_1$  have different directions, namely  $I_0$  is from the near end to the far end and  $I_1$  is from the far end to the near end. Solving the equations above we get,

$$V_0 \rightarrow -\frac{-2V_1 + e^{-ikl-\alpha}I_0Z - e^{ikl+\alpha}I_0Z}{e^{-ikl-\alpha} + e^{ikl+\alpha}}$$

$$I_1 \rightarrow -\frac{e^{-ikl-\alpha}V_1 - e^{ikl+\alpha}V_1 + 2I_0Z}{(e^{-ikl-\alpha} + e^{ikl+\alpha})Z}$$

Resulting in the following equations for the bi-port,

$$\begin{pmatrix} A_v & Z_0 \\ G_1 & A_g \end{pmatrix} = \begin{pmatrix} \frac{2e^{l(ik+\alpha)}}{1 + e^{2l(ik+\alpha)}} & \frac{(-1 + e^{2l(ik+\alpha)})Z}{1 + e^{2l(ik+\alpha)}} \\ \frac{-1 + e^{2l(ik+\alpha)}}{(1 + e^{2l(ik+\alpha)})Z} & -\frac{2e^{l(ik+\alpha)}}{1 + e^{2l(ik+\alpha)}} \end{pmatrix}$$

With,

$$\begin{pmatrix} V_0 \\ I_1 \end{pmatrix} = \begin{pmatrix} A_v & Z_0 \\ G_1 & A_g \end{pmatrix} \begin{pmatrix} V_1 \\ I_0 \end{pmatrix}$$

And

$$V_0 = A_v V_1 + Z_0 I_0$$

We can also write,

$$V_0 \rightarrow -\frac{-e^{-ikl-\alpha}I_0Z - e^{ikl+\alpha}I_0Z - 2I_1Z}{-e^{-ikl-\alpha} + e^{ikl+\alpha}}$$

$$V_1 \rightarrow -\frac{-2I_0Z - e^{-ikl-\alpha}I_1Z - e^{ikl+\alpha}I_1Z}{-e^{-ikl-\alpha} + e^{ikl+\alpha}}$$

$$A \rightarrow -\frac{e^{ikl+\alpha}I_0Z + I_1Z}{e^{-ikl-\alpha} - e^{ikl+\alpha}}$$

$$B \rightarrow \frac{(e^{-ikl-l\alpha}I_0 + I_1)Z}{-e^{-ikl-l\alpha} + e^{ikl+l\alpha}}$$

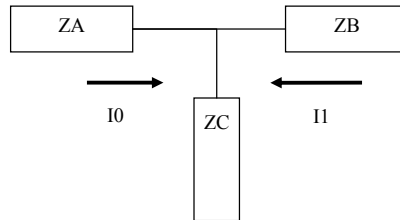
Resulting in,

$$\begin{pmatrix} Z_{00} & Z_{01} \\ Z_{10} & Z_{11} \end{pmatrix} = \begin{pmatrix} \frac{(1 + e^{2l(ik+\alpha)})Z}{-1 + e^{2l(ik+\alpha)}} & \frac{2e^{l(ik+\alpha)}Z}{-1 + e^{2l(ik+\alpha)}} \\ \frac{2e^{l(ik+\alpha)}Z}{-1 + e^{2l(ik+\alpha)}} & \frac{(1 + e^{2l(ik+\alpha)})Z}{-1 + e^{2l(ik+\alpha)}} \end{pmatrix}$$

With

$$\begin{pmatrix} V_0 \\ V_1 \end{pmatrix} = \begin{pmatrix} Z_{00} & Z_{01} \\ Z_{10} & Z_{11} \end{pmatrix} \begin{pmatrix} I_0 \\ I_1 \end{pmatrix}$$

Fitting this model to a three impedance model as the one shown in the figure requires the use of negative resistances.



## Useful frequency band

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Ao contrário do que se pode a primeira vista pensar temos que em geral o que limita a frequência de funcionamento de uma linha de transmissão não é um filtro passa baixo.

Para baixas frequências uma linha pode de facto simular um filtro passa baixo composto por uma bobina e um condensador, mas tal pode simplesmente ser interpretado com reflexões na linha.

Para frequências maiores isso reflectesse apenas como nulos na função de transferência da linha (estes podem ser eliminados se a linha estiver adaptada de forma a evitar reflectões).

Na prática o que limite a frequência máxima que normalmente se poderia utilizar numa linha é o aparecimento de modos que não o TEM. O modo TEM pode ser interpretado como uma onda que se propaga em paralelo com o guia (o campo eléctrico e o magnético não têm componente transversal, ou seja perpendicular a direcção de propagação). Outros modos podem ser interpretados como modos que vão as zig zags ao longo do guia e portanto a uma velocidade diferente.

O surgimento de modos para além do TEM provoca que diferentes ondas se propagem a diferentes velocidades ao longo do guia resultando em interferência inter simbólica. Num cabo coaxial RG-400 por exemplo isto ocorre nos 31.5 GHz.

No entanto acima desta frequência ainda existe propagação na linha.

## Models for the coupling of the power line channel

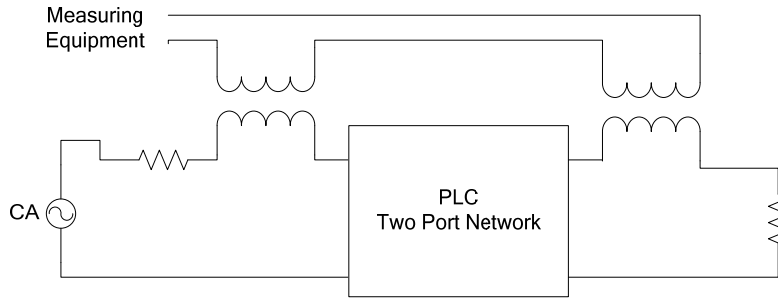
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The coupling between the power line channel and external equipment should be mostly inductive. Capacitive coupling should be low since the distances between the equipment are too large to form reasonable capacitors.

This means that in order to reduce coupling the induced current must be minimized. The values of the coupling voltage will depend on the value of the voltage drop across the transformer that models the coupling between the PLC channel and other circuits. The interference will appear to the external circuit as a noise source with a Thevenin given by a voltage and impedance, where the voltage is proportional to the voltage drop at the



transformer. The coupling factor is related to the amount of magnetic flux created by the power line loop that couples with the measuring equipment. The transformer will reflect the resistance in the outer circuit to the PLC network. The goal is then to minimize the voltage drop at the reflected resistance, which is equivalent to minimize the current. The position of the transformer in the circuit is only relevant because of changes in the current by leaks to ground.



The attenuation of the signals through the power line represented by the two port network means that the impedance seen at its input may be closer to the transmission line characteristic impedance than to the charge impedance.

If the external circuit is open or has high impedance then the voltage drop across the transformer will be given by the self induction of the line. The self induction,  $L$ , relates the magnetic flux with the current at the line,  $\Phi = L I$ . The self induction of a length,  $l$ , and diameter,  $d$ , conductor is given by,

$$L = l \left( \log \left( \frac{4l}{d} - 1 \right) \right) 200 \times 10^{-9}$$

For a 100m, 1mm conductor  $L$  is 0.257984mH resulting in 16j k $\Omega$  impedance at 10MHz however in a PLC line the value of the impedance can oscillate since for instance the magnetic flux created by phase and neutral on the line can subtract. Also considering the full length of the line may not be the best option since only a fraction of this magnetic flux will couple with the external circuit.

The radiation from is also mostly due to the current value. The Maxwell equations can be written using the vector potential field,  $\mathbf{A}$ , and the electric potential  $\Phi$ . Resulting in,

$$\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla^2\Phi + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}) = -\frac{\rho}{\epsilon_0}$$

$$\left(\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}\right) - \nabla\left(\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right) = -\mu_0\mathbf{J}.$$

where the vector potential field,  $\mathbf{A}$  is only due to currents sources.

# Adaptive impedances

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It may be difficult to improve performance by using adaptive impedance at the receiver, but controlling high injected current at the emitter when the input impedance is low seems promising.

## Adaptive receiver impedance

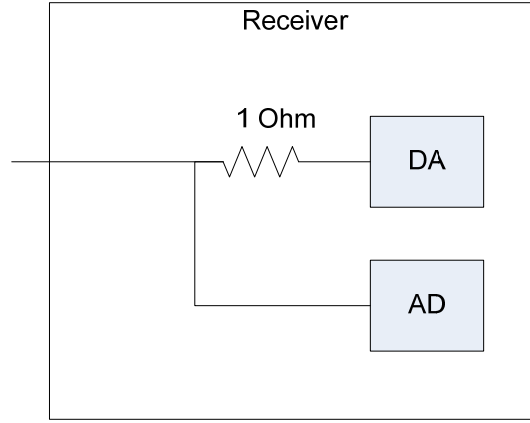
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Although the impedance of the noise source and the signal source (the emitter) are in general different the Thévenin equivalent of both sources seen at the receiver are the same. This means that the signal to noise ratio at the receiver will be independent of the charge impedance. This will be given for a voltage signal source and by,

$$Signal/Noise = \frac{R_n V_s}{R_s V_n}$$

However, it is expected that the noise source be less able to inject high power than the emitter, and so that the noise impedance increases if a low impedance is used in the receiver. **This may be difficult in practice.** This lowers the measured signal level and a trade off must be solved.

Frequency dependent adaptive impedance can be implemented at the receiver by making use of the digital to analogue converter already present to inject a voltage that simulates the effect of different impedance. This may increase or decrease the impedance already present.



Significant operation point modifications may occur in the noise source since the noise may be from frequencies not related to the operation of the device.

### *Modifying receiver impedance*

The signal injected by the DA converter can only be a delayed version of the signal measured by the AD. Assuming that this signal is the AD measured signal multiplied by a constant this will result in,

$$I_{in}(t) = \frac{V_{in}(t) - V_{DA}(t)}{Rc} = \frac{V_{in}(t) - k V_{In}(t - T)}{Rc}$$

where T is the sampling frequency, or using the Z transform,

$$I_{in}(z) = \frac{V_{in}(z)(1 - k z^{-1})}{Rc}$$

and,

$$Z(z) = \frac{Rc}{(1 - k z^{-1})}$$

If k is much greater than one (let's say ten), and since we are mostly interested in the magnitude, one has,

$$|Z(z)| = \frac{Rc}{k}$$

Not however that the AD may have a delay much greater than one. The delay will add to the impulse response of the system, and must be lower than the circular prefix in a OFDM system.

It is not easy to make this frequency dependent. This requires the calculation of the Fourier transform of the received signal. This can be done using the DFT and resulting in a delay of at least N samples where N is the DFT length,

$$I_{in}(z) = \frac{V_{in}(z)(1 - k(z) z^{-N})}{R_c}$$

The resulting delay will also add to the impulse response. This can be a large delay. In an OFDM system the delay could be similar to the symbol length and much greater than the circular prefix unless the N is small, and resolution is low. Frequency dependent impedance could be made smaller for large signal values and greater for lower signal values. This would keep the received signal at adequate levels (for the AD) while trying to increase noise impedance.

*The signal to noise ratio is independent of the receiver impedance.*

In many communications systems the emitter and receiver impedances are adapted to transmission systems in order to minimize reflections. However, it could be possible to achieve better signal to noise ratios if one tries to further use the power of the reflected signal. In the case of the receiver this is not possible, as shown below. In fact further reflections of a reflected wave do not depend if it is a signal or noise wave. **However, it may be possible to increase the noise impedance by decreasing the receptor impedance and increasing the current in the medium.** At the emission lower impedances should be better for the same voltage signal. Note however that coupling is mostly due to the current, so this suggests the use of a current source. **The receiver impedance should be a compromise between increasing the noise impedance and keeping the signal high for the AD.**

The signal to noise ratio is independent of the receiver impedance as long as the system is linear. In a non linear system it may be possible to increase the resistance of the noise source by changing the receiver resistance. Higher receiver impedance is better since it results in lower currents, lower radiation and bigger signals. However, it may be possible to improve signal to noise ratios by reducing the receiver impedance since it requires

more effort from the noise sources, and may lead to an increase in the noise sources impedance.

One has,

$$Signal = V_s \frac{R_n // R_c}{R_n // R_c + R_s}$$

$$Noise = V_n \frac{R_s // R_c}{R_s // R_c + R_n}$$

where,  $R_n$  is the noise resistance  $R_s$  is the emitter resistance and  $R_c$  is the charge resistance. Resulting in the signal to noise ratio,

$$Signal/Noise = \frac{R_n V_s}{R_s V_n}$$

That does not depend on  $R_c$ . Matched impedance maximizes the received power,

$$Received\ Power = V_s^2 \left( \frac{R_c}{R_c + R_s} \right)^2 / R_c$$

but may not maximize the signal to noise ratio. For a current source,

$$Signal = I_s (R_n // R_c) // R_s$$

$$Noise = V_n \frac{R_s // R_c}{R_s // R_c + R_n}$$

where  $I_s$  is the current source amplitude. Resulting in a signal to noise ratio of

$$Signal/Noise = \frac{I_s R_n}{V_n}$$

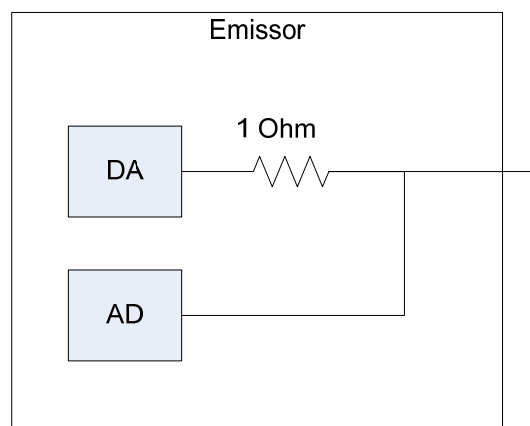
Current sources may generate high voltages that cause high interference with other equipment or saturate the amplifier. From the previous equation one gets that the

emitter impedance should be low, while for current sources it is high. Note however that this is for a given voltage signal, and that inductive coupling limits mostly the current signal and not the voltage.

# Emitter

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An inductive coupling indicates in order to reduce interference the current signal should be limited. Since one wished to maximize the signal this leads to the use of a constant average currents at the emitter (QAM or other), and may suggest the use of a current source instead of a voltage source. The same effect as the use of a current source is to select the voltage signal proportional to the circuit input impedance. This may be calculated using the AD converter also present at the emitter since it can also function as a receiver.



In fact in practice the voltage signal will also be limited, implying that the circuit input impedance must always be known. This implies that the signal amplitude at each frequency (that will be used in bit loading) will be given by,

$$V_{MAX} = \min\{|Z_{IN}|I_{MAX}, V_{GLOBAL MAX}\}.$$

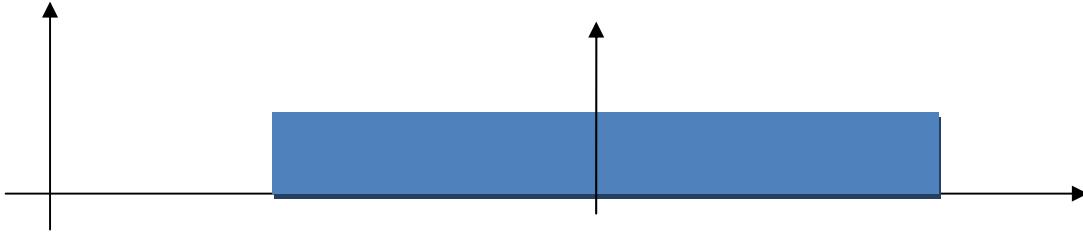
The voltage signal will interfere with other equipments throw capacitive coupling but this should be low since the distances between the equipment are too large to form reasonable capacitors.

As shown before the impedance of a transmission line oscillates between  $Z_c$ , the receiver impedance, and  $Z_0^2 / Z_c$ , were  $Z_0$  is the transmission line impedance. The length of the line is comparable to the wavelength for frequencies in the range of tens of MHz. If the transmission line were terminated by matched impedance the input impedance would be constant as in many communications systems, however in home power line networks several appliances may be connected to the line and impedance



matching is very difficult if not impossible. That means that the input impedance will vary. This can be result from the transmission line equations or from the different appliances in the line. **Low input impedances will limit the signal level since current is limited.**

#### *A note on a Complex signal demodulator*



Normally in order to demodulate a signal one like the one represented in the figure it should first be band-pass filterer then moved to a higher intermediate frequency and the moved back to low frequency and once again band pass filtered. This means that two filtering operations and an intermediate frequency would be required.

In the modem we propose the modulation by **multiplying by a complex sinusoidal** that moves the entire signal to base band as a hole. Namely we will multiply the signal by,

The signal  $s(t)$  will be multiplied by

$$\cos(\omega t) + \sin(\omega t) j$$

This results that the *real signal component* will be,

$$\cos(\omega t) = \frac{e^{j \omega t} + e^{-j \omega t}}{2}$$

And the real signal component will be,

$$r(t) \frac{e^{j \omega t}}{2} + r(t) \frac{e^{-j \omega t}}{2}$$

That is,

$$A + B$$

The complex real signal component will be,

$$\frac{A}{j} - \frac{B}{j}$$

And the resulting signal will be,

$$(A + B) + \left(\frac{A}{j} - \frac{B}{j}\right)j$$

So the B component will cancel out. This means that although the real and imaginary parts have unwanted signal components the resulting complex signal will not. Namely the point in the 2D complex plane used for the decisions will be at the correct position.

# Demodulation techniques in the presence of non Gaussian or non stationary noise

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More complex techniques non linear techniques as the ones proposed require the estimation of many parameters and may result impractical in many cases. A review of possible techniques is made, in order to try to reach a practical solution.

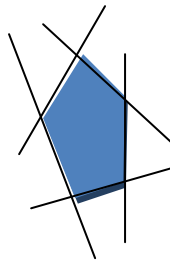
OFDM is good in avoiding narrow band noise, but bad with impulse noise, since it spread several frequencies. This may be reduced using correlation of the noise in between bands, maybe using interpolation. Even in the case of narrow band, correlation between bands may appear.

## *Time varying bit rate inside a packet*

As the number of modulations parameters increases the time required to estimate them also increases. One possibility is to slowly increase the binary rate as the system tunes to the channel. Similar effect would be to increase the binary rate as the error rate decreases.

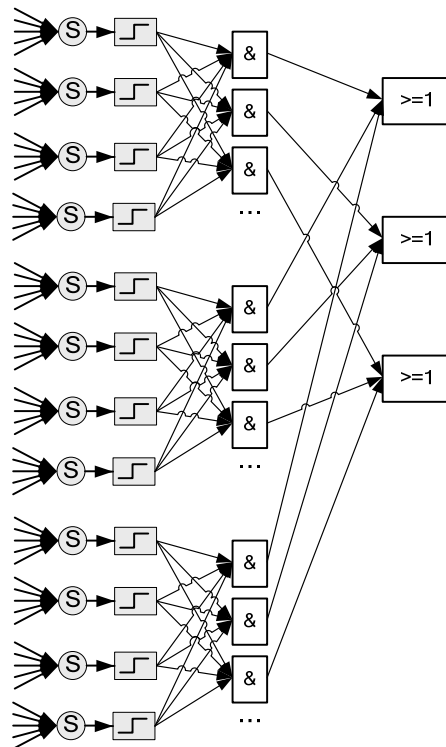
## *A demodulator based on neural network classifier or adaptive decision surfaces*

A neural network neuron can be viewed simply as function that divides the input space in two regions using a hyper-plane decision surface. Gathering neurons together allows one to make more complicated decision surfaces limited by sets of hyper-planes.



**Figure 1 – A complex decision surface limited by several hyper planes (straight lines in this case)**

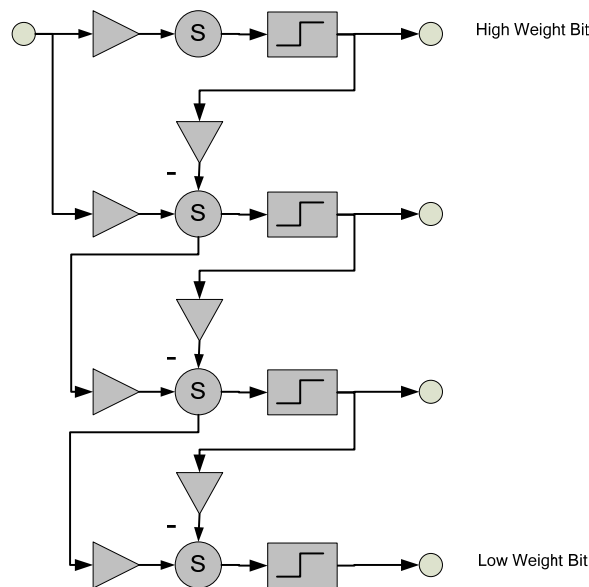
A technique like this may be used to make the demodulator of a communication system. In this case if the input signal value is in a given region will correspond to the reception of a given symbol. Something similar to this is done in QAM demodulation where the given the received value the symbol closest to it in the modulation is selected as the most likely transmitted signal. However, other decision surface may be used especially in the case of OFDM (orthogonal frequency division multiplexing) since in this case the received signal has more than two dimensions. An example of a complex demodulator based on a neural net with many uncorrelated decision surfaces is presented below.



**Figure 2 – A design for a demodulator/classifier based on a neural network, in a demodulation space defined by the signal at several sub bands or time slots. Non-gaussian PDF can results in more complicated decision surfaces. The connection to the AND gates can be simple or inverting.**

### *A brief look to a simple case like PAM*

Even a simple one-dimensional demodulation system like in PAM (Pulse Amplitude Modulation) may require a lot of different decision surfaces if the number of bits modulated is not very low. A full adaptive solution would require the determination of all this surfaces (in this case this are simple decision levels). And this may be costly to implement. A PAM demodulator can implemented using techniques similar to the ones used in AD conversion. A solution can be a successive approximation AD as show in the figure bellow. This greatly reduces the number of the decision surfaces.



For four bits this results in a four layer network where the inputs depend not only of the previous layer output but on all the previous layers and on the input. In this structure once the decision on the high weight bit is made the decision is similar for a high and for a low value. This may not be the case for non linear system with non Gaussian noise.

The general solution with independent decision surfaces is like a flash converter. The more practical option may be to use something like successive approximation to get a

first approximation that in many cases may not be too rude, and then refine it with the actual decision values chosen. Other possibility could be to reduce the number decisions by excluding some decisions surfaces based on previous decisions. Each decision could have associated a table with the set of outputs it eliminates and the algorithm would progress until only one valid output remained. A decision tree could also be interesting, were first a decision was made for the high weight bit. The decision surface for the next bit would be dependent of the previous decision and so forward. The problem in this case is that the first decision surface could be highly complex. **However all this solutions require the determination of a number of decision levels, and that may not be practical in practice.**

Other positions for the decision levels other than linearly spaced can be justified for instance if the noise level is correlated in the transmitted signal. If the noise is independent of the transmitted amplitude the decision point for the PAM signal will be in the middle of the transmitted values. This can be explained as follows. For noise with pdf equal to  $F$ , the likelihood of receiving symbol  $S_1$ , that has amplitude  $A_1$  is  $P(S_1)=F(X-A_1)$ . This should be equal to the likelihood of receiving symbol,  $S_2$ ,  $P(S_2)=F(X-A_2)$  at the decision point. For  $X$  in between  $A_1$  and  $A_2$   $F$  is usually invertible. This results in  $X = (A_1+A_2)/2$  as expected. However, for instance, if by some non-linear mechanism the noise level rises or decreases with the signal level, then a decision point different from the middle should be used.

### *The OFDM case*

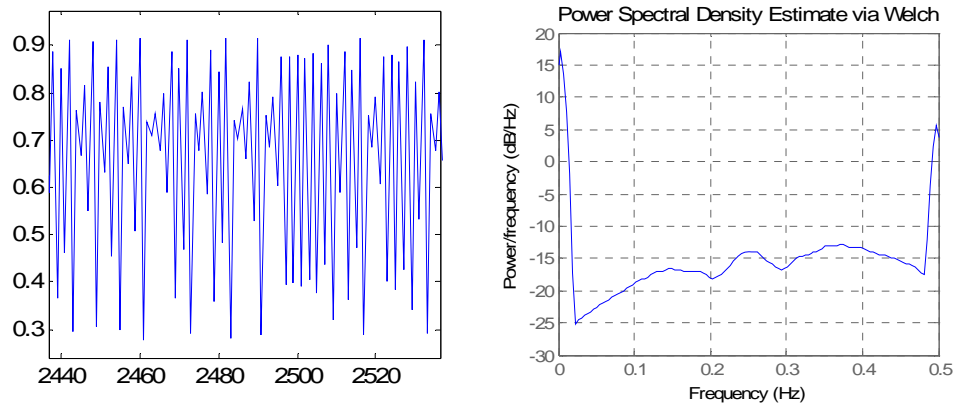
Adjusting decision levels because of variations of noise with signal level as in PAM may pay much. However, in OFDM the received signal is more complex since it is a vector and may give other opportunities. If the signal is a white stationary there is no correlation between the different values at the output of the DFT and this implies that each frequency should be demodulated independently.

However non Gaussian noise may be white but have a low embedding dimension, being the results of some chaotic phenomenon. And this may or not result in a signal that appears to be white, but that, the amplitude of the signal at different frequencies are

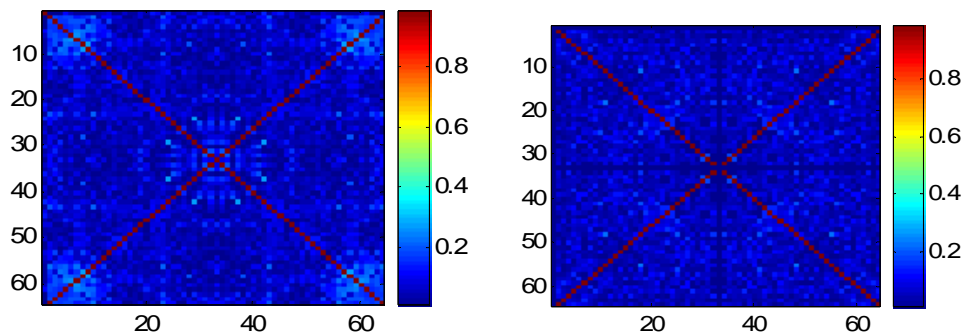
correlated. A very simple chaotic signal is a Logistic map (see List of chaotic maps in Wikipedia), with,

$$x(n+1) = r * x(n) * (1 - x(n)).$$

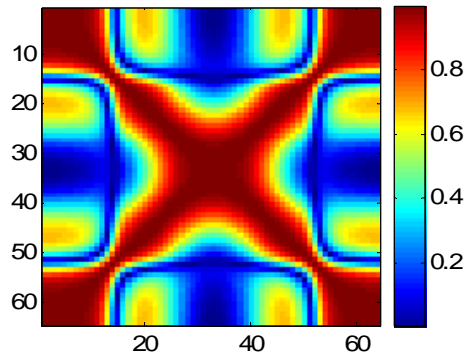
Running the Matlab script “LogisticMap.m” one gets the chaotic signal:



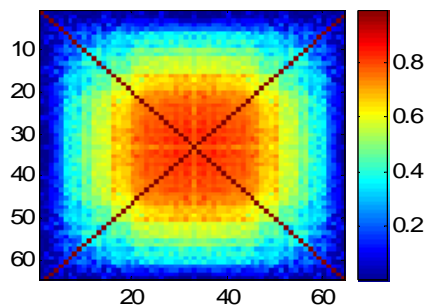
The script also analyses correlations between different FFT bins for the real and imaginary component as show in the following figures. Bellow is shown the case for  $r=3.77$ . There is no correlation between the real and imaginary parts.



For  $r=3.7400$  the signal becomes periodic, resulting in a high correlation in between bins.



For the circle map (“CircleMap.m”) for some values of the K appears a high correlation between the bins, bellow is an example for K=5.9.



A fairly simple case, were in which there is a high correlation in between bands, is for narrow band signals. This can be avoided if one adds a window and only choose some of the FFT bands. This should be much better than the original simple OFDM.

### *Why do more complicated decision surfaces can be useful?*

If one treats each band or time slot independently them one has to assume the worst case scenario value for each signal or random variable. The noise will consist of the variance of each random variable and this can be large even when the two variables are not independent and the actual amount of uncertainty is low. This will correspond to a low area occupied by the joint PDF. The dependence of the two variables can be linear, which correspond to a high correlation coefficient value but can also assume more complicated functions. Non independent noise signals will result more complex decision surfaces that maximise the posterior error probability, based in the bays formula.

### *Cross band noise canceller*



## *Impulse Noise*

OFDM is good in avoiding narrow band noise, but bad with impulse noise, since it spreads several frequencies. This may be reduced using correlation of the noise in between bands, maybe using interpolation. Even in the case of narrow band, correlation between bands may appear.

# Impulse noise

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Impulse noise spreads through several frequencies, and could be reduced by using correlation between bands and joint demodulation of the several OFDM receiver frequencies (just two or three) or through a cross band noise canceller.

For narrow impulses, impulse noise is more easily dealt with in time domain where forward error correction can be used to ignore error bits.